1. (a) Factorise completely $x^3 + 10x^2 + 25x$

(2)

(b) Sketch the curve with equation

$$y = x^3 + 10x^2 + 25x$$

showing the coordinates of the points at which the curve cuts or touches the x-axis.

(2)

The point with coordinates $(-3, 0)$ lies on the curve with equation

$$y = (x + a)^3 + 10(x + a)^2 + 25(x + a)$$

where a is a constant.

(c) Find the two possible values of a.

(3)

(a) $x^3+10x^2+25x = x(x^2+10x+25)$

$=x(x+5)(x+5)$

$=\boxed{x(x+5)^2}$

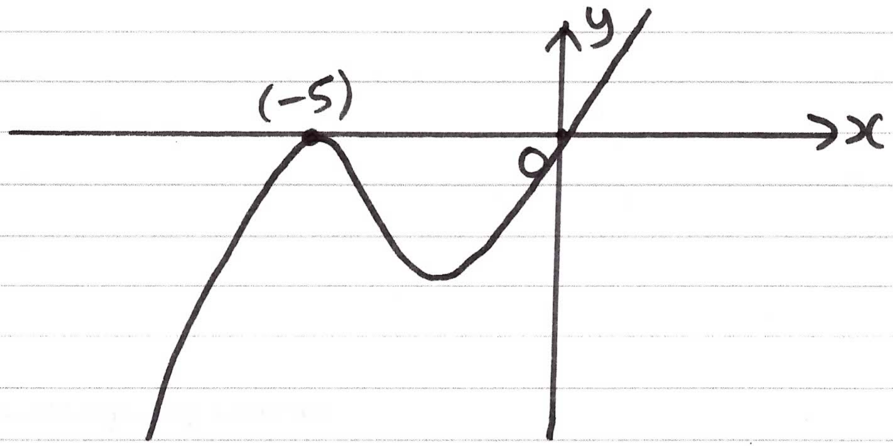
b) $y=x^3+10x^2+25x$

When the curve crosses the x-axis, $y=0$

$\therefore x^3+10x^2+25x=0$

$x(x+5)^2=0$

Either $x=0$ or $x=-5$ twice



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Question continued

(c) Let $x^3+10x^2+25x=f(x)$

Then $(x+a)^3+10(x+a)^2+25(x+a)$ is a transformation horizontally of '$-a$'.

If $(-3,0)$ lies on the curve $f(x+a)$, then the original x-intercept of $(-5,0)$ has been translated ~~verti~~ horizontally by '$+2$' or the x-intercept of $(0,0)$ has been translated horizontally by '-3'.

$\therefore \boxed{a=-2 \text{ or } a=3}$

(Total for Question is 7 marks)

2. $$g(x) = 4x^3 - 12x^2 - 15x + 50$$

(a) Use the factor theorem to show that $(x + 2)$ is a factor of $g(x)$.

(2)

(b) Hence show that $g(x)$ can be written in the form $g(x) = (x + 2)(ax + b)^2$, where a and b are integers to be found.

(4)

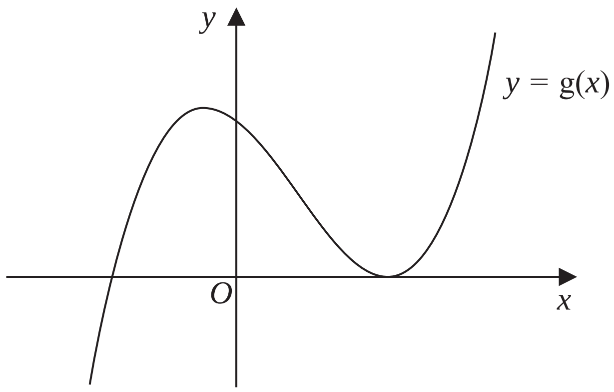

Figure 2

Figure 2 shows a sketch of part of the curve with equation $y = g(x)$

(c) Use your answer to part (b), and the sketch, to deduce the values of x for which

(i) $g(x) \leqslant 0$

(ii) $g(2x) = 0$

(3)

a) $(x+2) = 0 \quad x = -2$

$g(-2) = 4(-2)^3 - 12(-2)^2 - 15(-2) + 50$

$= 0$

$\therefore x+2$ is a factor

b)

$$\begin{array}{r}
4x^2 - 20x + 25 \\
x+2 \overline{) 4x^3 - 12x^2 - 15x + 50} \\
\underline{4x^3 + 8x^2} \\
-20x^2 - 15x \\
\underline{-20x^2 - 40x} \\
25x + 50 \\
\underline{25x + 50}
\end{array}$$

$g(x) = (x+2)(4x^2 - 20x + 25)$

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Question continued

c i) $(x+2)(4x^2-20x+25)=0$

$x=-2 \quad (2x-5)^2=0$

$x=\pm 2.5$

$x<-2 \quad x=2.5$

ii) $x=-1 \quad x=1.25$

3. $$f(x) = 2x^3 - 13x^2 + 8x + 48$$

(a) Prove that $(x - 4)$ is a factor of $f(x)$.

(2)

(b) Hence, using algebra, show that the equation $f(x) = 0$ has only two distinct roots.

(4)

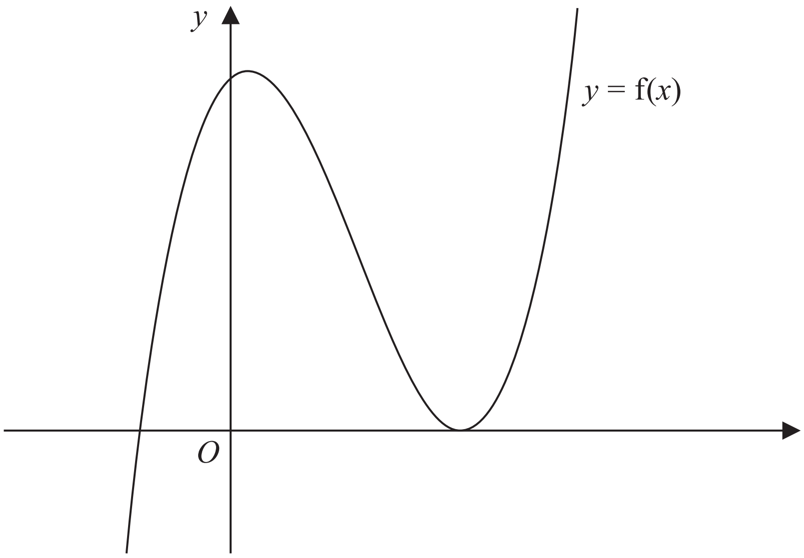

Figure 2

Figure 2 shows a sketch of part of the curve with equation $y = f(x)$.

(c) Deduce, giving reasons for your answer, the number of real roots of the equation

$$2x^3 - 13x^2 + 8x + 46 = 0$$

(2)

Given that k is a constant and the curve with equation $y = f(x + k)$ passes through the origin,

(d) find the two possible values of k.

(2)

a) $f(4) = 2(4)^3 - 13(4)^2 + 8(4) + 48$

$= 0$ // $\therefore$ $(x-4)$ is a factor

b) long division:

$$\begin{array}{r} 2x^2 - 5x - 12 \\ x-4 \overline{\smash{)}\, 2x^3 - 13x^2 + 8x + 48} \\ 2x^3 - 8x^2 \\ \hline 0 \quad -5x^2 + 8x \\ -5x^2 + 20x \end{array}$$

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Question **continued**

$$\begin{array}{r} 0 \quad -12x + 48 \\ -12x + 48 \\ \hline 0 \quad 0 \end{array}$$

//

so $f(x) = (x-4)(2x^2 - 5x - 12)$

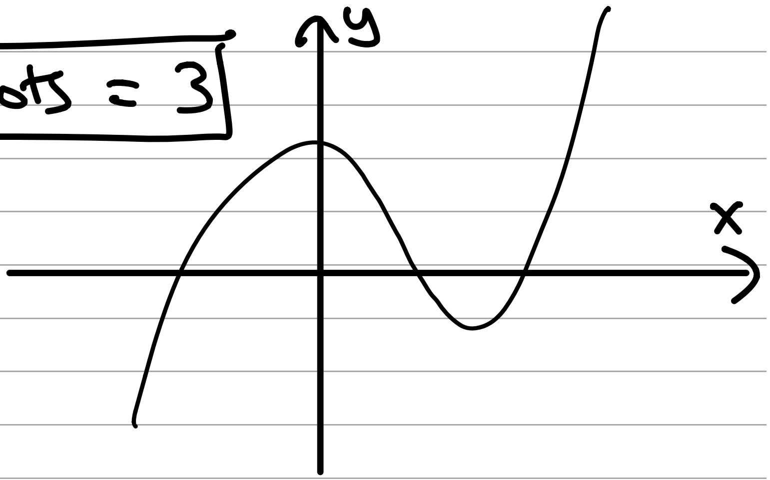
$= (x-4)(2x+3)(x-4)$

$= (x-4)^2(2x+3)$ //

hence $f(x)$ has just 2 distinct roots; $x = 4$
$x = -3/2$

c) The curve is shifted down by 2 units

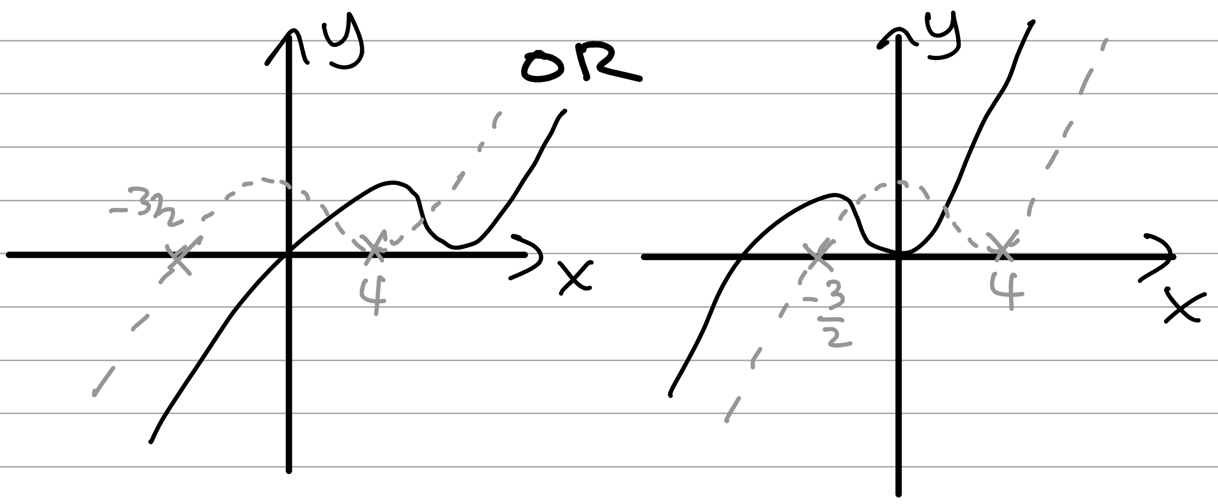
so $\boxed{\# \text{ of roots} = 3}$



Question **continued**

ORIGINAL

d) we could have :



f(x) shifted to the right by $3/2$

ie $f(x - 3/2)$

so $\boxed{k = -\frac{3}{2}}$

f(x) shifted to the left by 4

ie $f(x+4)$

so $\boxed{k = 4}$

4.

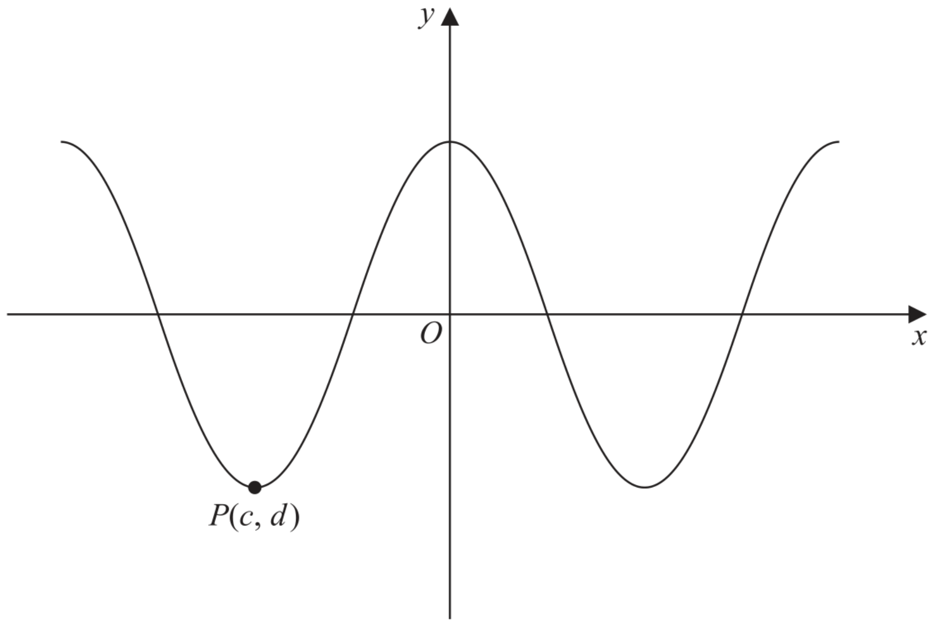


Figure 3

Figure 3 shows part of the curve with equation $y = 3\cos x°$.

The point $P(c, d)$ is a minimum point on the curve with c being the smallest negative value of x at which a minimum occurs.

(a) State the value of c and the value of d.

(1)

(b) State the coordinates of the point to which P is mapped by the transformation which transforms the curve with equation $y = 3\cos x°$ to the curve with equation

(i) $y = 3\cos\left(\frac{x°}{4}\right)$

(ii) $y = 3\cos(x - 36)°$

(2)

(c) Solve, for $450° \leqslant \theta < 720°$,

$$3\cos\theta = 8\tan\theta$$

giving your solution to one decimal place.

In part (c) you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

(5)

Question continued

a) minimum of $\cos x = -1 \Rightarrow$ minimum of $3\cos x = -3 = d$

P is the first minimum for $x<0$ $\therefore$ $c = -180°$

$P(-180°, -3)$

b) i. $y = 3\cos\left(\frac{x°}{4}\right) \Rightarrow$ x 'stretched' $\times 4$, no change to y

$\hookrightarrow (-720°, -3)$

ii. $y = 3\cos(x - 36°) \Rightarrow$ translation in x-direction $+36°$

$\hookrightarrow (-144°, -3)$

c) When solving these types of question, list the relevant trig. identities that could help you.

$\tan\theta \equiv \frac{\sin\theta}{\cos\theta}$, $\sin^2\theta + \cos^2\theta \equiv 1$

$3\cos\theta = 8\tan\theta$

$= 8\frac{\sin\theta}{\cos\theta}$

$\times \cos\theta$: $3\cos^2\theta = 8\sin\theta$

Form quadratic in $\sin\theta$ (you want an equation with only one trig. function)

$3(1 - \sin^2\theta) = 8\sin\theta$

Question continued

$$3\sin^2\theta + 8\sin\theta - 3 = 0$$

$$(3\sin\theta - 1)(\sin\theta + 3) = 0$$

$\sin\theta \neq -3$ so $\sin\theta = \frac{1}{3}$

always note what range you need

min. value is -1

$\theta = 19.47°$

but range is $450° \leq \theta < 720°$

$\theta = 180 - 19.47°$

$= 160.53°$

$\sin\theta$ repeats every $360°$

to bring into range: $160.53° + 360° = \underline{520.5°}$ (1 d.p.)

$19.47° + 360°$ also not in range

5.

$$f(x) = 2x^2 + 4x + 9 \qquad x \in \mathbb{R}$$

(a) Write f(x) in the form $a(x + b)^2 + c$, where a, b and c are integers to be found.

(3)

(b) Sketch the curve with equation $y = f(x)$ showing any points of intersection with the coordinate axes and the coordinates of any turning point.

(3)

(c) (i) Describe fully the transformation that maps the curve with equation $y = f(x)$ onto the curve with equation $y = g(x)$ where

$$g(x) = 2(x - 2)^2 + 4x - 3 \qquad x \in \mathbb{R}$$

(ii) Find the range of the function

$$h(x) = \frac{21}{2x^2 + 4x + 9} \qquad x \in \mathbb{R}$$

(4)

a) $f(x) = 2x^2 + 4x + 9$

$= 2(x^2 + 2x) + 9$ (1)

$= 2[(x+1)^2 - 1] + 9$ (1)

$= 2(x+1)^2 - 2 + 9$

$= 2(x+1)^2 + 7$ (1)

b) $y = 2(x+1)^2 + 7$

y intercept when $x = 0$

$y = 2(0+1)^2 + 7$

$= 2 + 7$

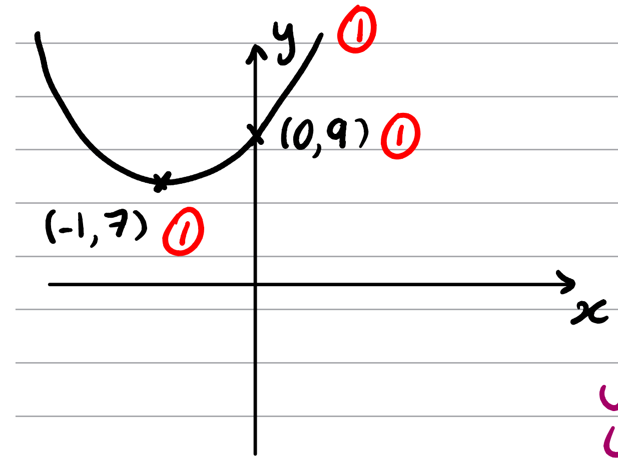
$= 9 \quad \therefore$ y intercept $(0,9)$

x intercept when $y = 0$

$0 = 2(x+1)^2 + 7$

$2(x+1)^2 = -7$

$(x+1)^2 = -\frac{7}{2}$ ✓ curve doesn't intercept x axis (no real roots)



Turning Point

$y = (x-a)^2 + b$ turning point (a,b)

$y = 2(x - -1)^2 + 7 \quad \therefore$ turning point $(-1, 7)$

c) i) $f(x) = 2x^2 + 4x + 9$
$g(x) = 2(x-2)^2 + 4x - 3$

$f(x-a)$ translation by vector $\begin{pmatrix} a \\ 0 \end{pmatrix}$

$$f(x-2) = 2(x-2)^2 + 4(x-2) + 9$$
$$= 2(x-2)^2 + 4x - 8 + 9$$
$$= 2(x-2)^2 + 4x + 1$$

$f(x)+b$ translation by vector $\begin{pmatrix} 0 \\ b \end{pmatrix}$

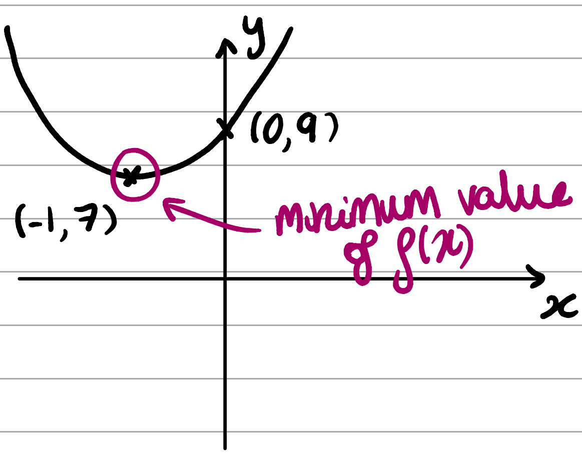
$$f(x-2) - 4 = 2(x-2)^2 + 4(x-2) + 9 - 4$$
$$= 2(x-2)^2 + 4x - 8 + 9 - 4$$
$$= 2(x-2)^2 + 4x - 3$$

$g(x) = f(x-2) - 4$

↖ a ↖ b

∴ The transformation that maps $y = f(x)$ onto $y = g(x)$ is a translation by vector $\begin{pmatrix} 2 \\ -4 \end{pmatrix}$ (2)

c) ii)



$$h(x) = \frac{21}{2x^2 + 4x + 9}$$
$$= \frac{21}{f(x)}$$

$f(x) \to \pm\infty$

$h(x) \to 0$

Maximum $h(x)$ when $f(x)$ is at its minimum

$$\frac{21}{7} = 3 \quad (1)$$

$0 < h(x) \leq 3$ (1)